Naval Research Laboratory
Washington, DC 20375-5000

NRL Memorandum Report 6752

The Impact of the Three-Wave Instability on the Spiral Line Induction Accelerator

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November 23, 1990

REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	1990 November 23	

4. TITLE AND SUBTITLE

The Impact of the Three-Wave Instability on the Spiral Line Induction Accelerator

5. FUNDING NUMBERS

ARPA Order #4395, A86
JO # 47-3593-00

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
Code 4790
Washington, DC 20375-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

NRL Memorandum Report 6752

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

DARPA
Arlington, VA 22209

NSWC
Silver Spring, MD 20903-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The solenoidal and helical quadrupole magnetic fields that are used to transport the high-current beam in the spiral line induction accelerator may admit instabilities such as the electromagnetic three-wave instability. Previous studies have shown that stability is easily maintained for energies up to $\simeq$ 40 MeV. ~~We find that~~ three-wave growth rates for energies exceeding 40 MeV can be $\lesssim 10^{-3}$ cm^{-1}. Strategies are outlined for stabilizing or further reducing growth rates. These are 1) cyclotron detuning, where the instability is detuned by varying the axial magnetic field, 2) damping the instability through the use of a low-Q beam pipe, where "low-Q" can include Q as high as 40,000 and 3) use of discrete quadrupole focusing, which allows the instability to be detuned by varying the quadrupole wavenumber. The effectiveness of de-Qing and cyclotron detuning is discussed for the various instability regimes.

14. SUBJECT TERMS

High current
Accelerator
Strong focusing

15. NUMBER OF PAGES 16

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std Z39-18
298-102

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THE IMPACT OF THE THREE-WAVE INSTABILITY ON THE SPIRAL LINE INDUCTION ACCELERATOR

I. INTRODUCTION

It has long been known that the strong focusing magnetic fields that are used to transport the high-current beam in the spiral line induction accelerator (SLIA)[1], may admit instabilities such as the three-wave instability.[2-4] The SLIA includes solenoidal focusing in the straight sections and both solenoidal and stellarator (helical quadrupole) focusing in the curved sections. The stellarator fields are included to focus the space charge of the beam over a large energy bandwidth.

Recent results have shown that there are several stability regimes for the three-wave instability[3,4] and that the SLIA proof-of-concept experiment (PoCE) can operate entirely in the stable regime. What has been less clear is the significance of the instability as the beam energy approaches 50 MeV. In the present paper we will address the difficulties associated with designing a three-wave stable SLIA with a peak beam energy of 50 MeV.

The three-wave instability is a resonant interaction between the beam centroid motion in the external magnetic fields and the TE_{11} mode of the beam pipe. The external fields include the axial field $B_z = B_o$ and the quadrupole field:

$$B_{qx} = -B_q k_q [x \sin(k_q z) - y \cos(k_q z)] \tag{1a}$$

$$B_{qy} = B_q k_q [x \cos(k_q z) + y \sin(k_q z)] \;, \tag{1b}$$

where $B_q k_q$ is the quadrupole gradient and k_q is the quadrupole wavenumber. For $\gamma >> 1$ the stability condition of Ref. 4 may be written as

$$K_o > K_c \equiv \frac{k_q}{2} + \frac{1}{2}\left[\left(k_q - \frac{2\mu_{11}}{\gamma}\right)^2 + \frac{4\gamma k_q^2 K_q^{\,2}}{\mu_{11}(k_q - \mu_{11}/\gamma)}\right]^{1/2} , \tag{2}$$

Manuscript approved September 18, 1990.

where $\gamma = (1 - \beta^2)^{-1/2}$ is the usual relativistic factor for the beam, $K_o = eB_o/\gamma\beta mc^2$ is the cyclotron wavenumber, $K_q = eB_q/\gamma\beta mc^2$, $\mu_{11} = 1.84/r_g$ is the cut-off frequency of the TE_{11} waveguide mode and r_g is the radius of the beam pipe. Note that for $K_q << (k_q\mu_{11}/4\gamma)^{1/2}$, Eq. (2) reduces to the stability condition of Ref 3. For example, with $\gamma = 90$, $k_q = 0.048\ cm^{-1}$ and $r_g = 3$ cm, parameters typical of a high energy SLIA bend[5], the stability condition of Ref. 3 is valid only for very small quadrupole gradients, $B_qk_q << 67$ G/cm.

The stability condition Eq. (2) gives a lower limit on K_o for given values of γ, k_q, B_qk_q and r_g such that as the beam energy increases, greater values of $B_o > 0$ are required for stability. With the complete set of stability conditions of Ref. 4, we obtain a stability diagram as shown in Fig. 1. Here, stable and unstable regions of (B_o,k_q) parameter space are delineated for given values of γ, B_qk_q, and r_g. In Fig. 1, $\gamma = 7$, $B_qk_q = 200$ G/cm and $r_g = 3$ cm. Note first that the SLIA PoCE will operate in the stable region that appears for $B_o > 0$ in Fig. 1. Note also that in our convention, $k_q > 0$, indicating a stellarator field with a right-handed pitch. For a left-handed stellarator field, $k_q < 0$, stable operation requires $B_o < 0$.

The difficulty of satisfying Eq. (2) at high energy can be seen in the stability diagram, shown as Fig. 2, for $\gamma = 90$, $B_qk_q = 73.4$ G/cm, and $r_g = 3$ cm. Figure 2 shows that $B_o > 8.5$ kG would be required for stability. This is confirmed by numerical solutions of the dispersion relation. Because such high values of B_o may not be desirable and because three-wave growth rates fall as $\gamma^{-1/2}$, it may be favorable to operate in a mildly unstable regime at high energy. Typical mildly unstable parameters for a $\gamma = 90$ SLIA bend are shown in Table I. Our uncertainty as to how much growth is

tolerable suggests to us that we consider alternate strategies for reducing or eliminating growth in this regime.

In the present paper we discuss three such strategies. In section II we discuss detuning the instability by varying the axial guide field. Section II contains a discussion of the possibility of damping the instability through the use of a low-Q beam pipe. In section IV, we consider the possible use of a discrete alternating gradient quadrupole focussing system, also known as a FODO lattice. This opens up the possibility of detuning the instability by varying the quadrupole wavenumber. We summarize our results in section V.

II. DETUNING IN THREE-WAVE UNSTABLE REGIME II

Recent ELBA[6] simulation results showed saturation of the instability in three-wave unstable regime II ($k_q/2 + 2K_q < K_o \leq K_c$). Unstable regime II corresponds to unstable region II of (B_o,k_q) parameter space as shown in Figs. 1 and 2. This saturation, which occurred without emittance growth or current loss, was the result of detuning.[7] In this scenario, the beam loses energy to the TE_{11} waveguide mode and, at the same time, the parallel velocity of the beam is reduced as the transverse motion increases. Both of these effects increase the cyclotron wavenumber K_o such that the three-wave interaction is detuned. This detuning changes the frequency ω of the unstable mode. The transverse beam centroid motion, which has wavenumber $\omega/\beta c$, cannot respond promptly to this frequency shift with the result that the beam motion is damped.

Diagnostics of beam energy and axial and parallel velocity of the beam centroid from several ELBA runs that exhibited this saturation showed that both γ and β_z decreased by several percent as the instability grew.[7] This

shifted the cyclotron wavenumber by ≈10% from the point when the instability began to grow, which was taken to be one-third of the saturation distance, to the point of saturation. Solutions of the dispersion relation for those parameters showed that for a 10% change in either B_o or γ such that K_o increased by 10%, the unstable interaction point in (ω,k) space shifted so as to stabilize the previous region (in frequency space) of peak growth.

Analysis confirms that detuning in K_o is effective in three-wave unstable regime II but has little or no effect in regime I ($K_o < k_q/2 - K_q$). To see this, we must reexamine the three-wave interaction. Here we consider unstable waveguide modes with $\omega > 0$, but the discussion carries easily to the $\omega < 0$ case. As one sweeps through the possible values of K_o, the equation of the unstable beam mode in (ω,k) space in the $I_b \to 0$, $K_q \to 0$ limit changes.[4] For $K_o \leq 0$ (in unstable regime I), the equation of the beam mode is

$$k \approx \omega/c - k_q \; . \tag{3a}$$

For $0 < K_o < k_q/2$ (also in regime I), the unstable mode is

$$k \approx \omega/c + K_o - k_q \; . \tag{3b}$$

For $k_q/2 < K_o < k_q$ (unstable regime II), the unstable mode is again given by

$$k \approx \omega/c + K_o - k_q \; . \tag{3c}$$

For $K_o > k_q$, the beam is stable in the $I_b \to 0$, $K_q \to 0$ limit.

We see that in unstable regime I, shifts in K_o have little effect because the beam mode is is either independent of K_o or restricted to $K_o < k_q/2$. In unstable regime II, detuning is possible and is observed.

These results suggest that it may be possible to operate with low growth rates in three-wave unstable regime II by employing cyclotron detuning. Here one periodically shifts the magnitude of the axial guide field to shift the instability from frequency to frequency, thus keeping transverse beam motion at any given frequency below significant levels. Numerical solutions of the dispersion relation show that the latter two cases of Table I can be detuned with shifts in B_o of <10%.

A refinement of this detuning technique might include tuned stubs at the location of each change in the axial magnetic field value. In theory, these could be adjusted so as to reflect the low level TE_{11} mode at the point where the unstable frequency is shifted and a new mode begins to grow.

Note that Eqs. (3a-c) show that detuning the instability via changes in k_q, rather than K_o, may be possible. Results for alternating gradient quadrupole focusing systems, presented in section IV below, confirm this possibility. Fabricating a helical quadrupole magnet with a varying pitch length, however, is considered prohibitively expensive.

III. DAMPING VIA LOW Q

Because growth rates in the high energy regime are small, it may be possible to damp the instability by lowering the Q of the beam pipe. If we assume that the presence of damping does not change the nature of the instability, the three-wave interaction will be stabilized when

$$Q \leq \omega/2\Gamma_\infty \tag{4}$$

where ω is the frequency of the unstable waveguide mode and Γ_∞ is the growth rate from the linear dispersion relation with $Q = \infty$. Analytical results[8] in which damping was added to the dispersion relation have verified Eq. (4).

For example, for the three parameter sets of Table I, the instability can be damped with the rather high Q values of 850, 3700 and 40,000 respectively.

Another example is the NRL modified betatron accelerator (MBA) in which the addition of strong focusing fields has aided the successful acceleration of a 1 kA electron beam through >38000 turns in the 100 cm major radius device to obtain 15 MeV.[9] Typical MBA parameters are beam current $I_b = 1$ kA, $B_o = -2$ kG, $B_q k_q = 20$ G/cm, $k_q = 0.06\ \mathrm{cm}^{-1}$ and $r_g = 15$ cm. As the beam is accelerated from 5 to 15 MeV, the growth rate varies from $\Gamma_\infty/c = 5.10 \times 10^{-4}\ \mathrm{cm}^{-1}$ to $3.04 \times 10^{-4}\ \mathrm{cm}^{-1}$. Because the frequency of the unstable mode is insensitive to changes in the cyclotron wavenumber in this regime, the unstable frequency remains constant at $\omega/c = 0.16\ \mathrm{cm}^{-1}$ ($\omega = 4.8$ GHz). Here $Q \leq 150$ is required for stability. The MBA vacuum chamber has $Q \simeq 150$. Three-wave stability was confirmed by radiation measurements using a broad-band antenna which showed radiation levels not exceeding those expected from single-particle emission.[10]

IV. DISCREET QUADRUPOLE FOCUSING

Discrete quadrupole focusing systems, also called FODO lattices, have stability properties that differ from helical quadrupole systems.[11] The focusing field for such a system may be approximated by

$$B_{qx} = -B_q k_q \cos(k_q z) y , \qquad B_{qy} = -B_q k_q \cos(k_q z) x \tag{5}$$

where k_q is the FODO wavenumber, $B_q k_q$ is the FODO gradient, z is the axial coordinate and x and y are transverse coordinates. Note that the FODO lattice field has no preferred direction so that we expect $B_o < 0$ to be equivalent to $B_o > 0$. In fact, when the dispersion relation is solved and approximate stability boundaries are found[11], we obtain the stability diagram shown in Fig. 3. Here, $\gamma = 7$, $B_q k_q = 200$ G/cm and $r_g = 3$ cm.

It is clear from the stability diagram that the overall stability properties are not as favorable as in the helical quadrupole case. However, we find that for identical values of I_b, γ, $B_q k_q$ and k_q, peak growth rates are lower, typically by $2^{1/2}$, in the FODO configuration. Furthermore, the flexibility of the FODO system allows growth rates to be reduced via detuning in the FODO wavenumber k_q.

We find that the FODO wavenumber can be changed so as to shift the unstable mode from frequency to frequency. Also, this detuning mechanism is applicable to low axial magnetic fields $(0 \leq K_o < k_q - 2K_q)$ where cyclotron detuning is not effective. This is demonstrated in Fig. 4, which shows growth rate versus frequency for a FODO lattice with $B_o = 0$, $B_q K_q = 200$ G/cm and two different values of $k_q = 0.4$ and 0.6 cm^{-1}. Here, $I_b = 10$ kA, $\gamma = 7$ and $r_g = 3$ cm. We see that the unstable range at $k_q = 0.6$ cm^{-1} is completely stabilized when the FODO wavenumber is shifted to $k_q = 0.4$ cm^{-1}. As with the helical quadrupole case, detuning will be effective if the beam centroid wavenumber, $\omega/\beta c$, is changed often enough that significant transverse beam motion does not develop. This scenario may be complicated by the multiple unstable wavenumbers associated with the unstable frequency of the FODO system, making it difficult to detune the TE_{11} mode both in frequency and wavenumber as can be done in the helical quadrupole case. We note, however, that the importance of this is not well understood.

Furthermore, designing such a minimum-growth FODO system may be somewhat time and computing intensive. Note that the instability in this configuration has been verified using the ELBA simulation code.[11]

V. CONCLUSIONS

Previous results have shown that three-wave stability in the SLIA is easily maintained for energies up to ≃40 MeV. We find that growth rates for energies exceeding 40 MeV can be $\lesssim 10^{-3}$ cm^{-1}. It may be possible to determine whether growth rates of order 10^{-3} cm^{-1} are permissible over the last 2-3 bends of a 50 MeV SLIA via PoCE experiments. If such growth rates are not acceptable, we have outlined three strategies for stabilizing or reducing the three-wave interaction. These are 1) cyclotron detuning, wherein the instability is detuned over the length of a given bend by varying the axial magnetic field, 2) damping the instability through the use of a "low-Q" beam pipe and 3) discrete quadrupole focusing, for which it is feasible to detune the instability by varying the discrete quadrupole wavenumber over the length of the curve. We show that "low-Q" can include Q as high as 40,000. Note that de-Qing and cyclotron detuning are most effective in thee-wave unstable regime II, where the interaction frequency is highest and is sensitive to the cyclotron wavenumber.

Acknowledgments

This work was supported by Defense Advanced Research Projects Agency, ARPA Order Number 4395, Amendment 86, monitored by Naval Surface Warfare Center. We are also grateful to D. Chernin, T. Hughes, P. Sprangle, G. Joyce and S. Putnam for many illuminating discussions

References

1. A. Mondelli, D. Cher: -, S. D. Putnam, L. Schlitt and V. Bailey, Proc. Sixth Intl. Conf. on High Power Particle Beams (Osaka, Japan), (1986); also V. Bailey, L. Schlitt, M. Tiefenback, S. Putnam, A. Mondelli, D. Chernin and J. Petillo, Proc. 1987 IEEE Part. Accel. Conf., 920 (1987).

2. T. P. Hughes and B. B. Godfrey, Phys. Fluids 29, 5 (1986).

3. D. Chernin and T. P. Hughes (private communication)

4. C. M. Tang, P. Sprangle, J. Krall, P. Serafim and F. Mako, Part. Accel. (in press).

5. S. Putnam (private communication).

6. G. Joyce, J. Krall and S. Slinker, Proc. of the Conf. on Computer Codes and the Linear Accel. Community, LANL Report LA-11857-C, 341 (1990).

7. J. Krall, C. M. Tang, G. Joyce and P. Sprangle, Phys. Fluids B (December, 1990).

8. T. Hughes (private communication).

9. C. A. Kapetanakos, L. K. Len, T. Smith, J. Golden, K. Smith, S. J. Marsh, D. Dialetis, J. Mathew, P. Loschialpo, J. H. Chang, Phys. Rev. Lett. 64, 2374 (1990).

10. C. A. Kapetanakos (private communication).

11. C. M. Tang, J. Krall, D. Chernin and T. Hughes, submitted to Phys. Rev. D.

Table I

B_o (kG)	$B_q k_q$ (G/cm)	k_q (cm^{-1})	ω/c (cm^{-1})	Γ/c (cm^{-1})
-6.91	235.0	0.064	3.41	2.0×10^{-3}
7.10	73.4	0.048	11.23	1.5×10^{-3}
7.71	58.3	0.048	28.56	3.5×10^{-4}

Unstable growth rates and frequencies for typical SLIA achromat parameters[5] with $\gamma = 90$, $I_b = 10$ kA, $r_g = 3$ cm and bend radius $R_o = 250$ cm.

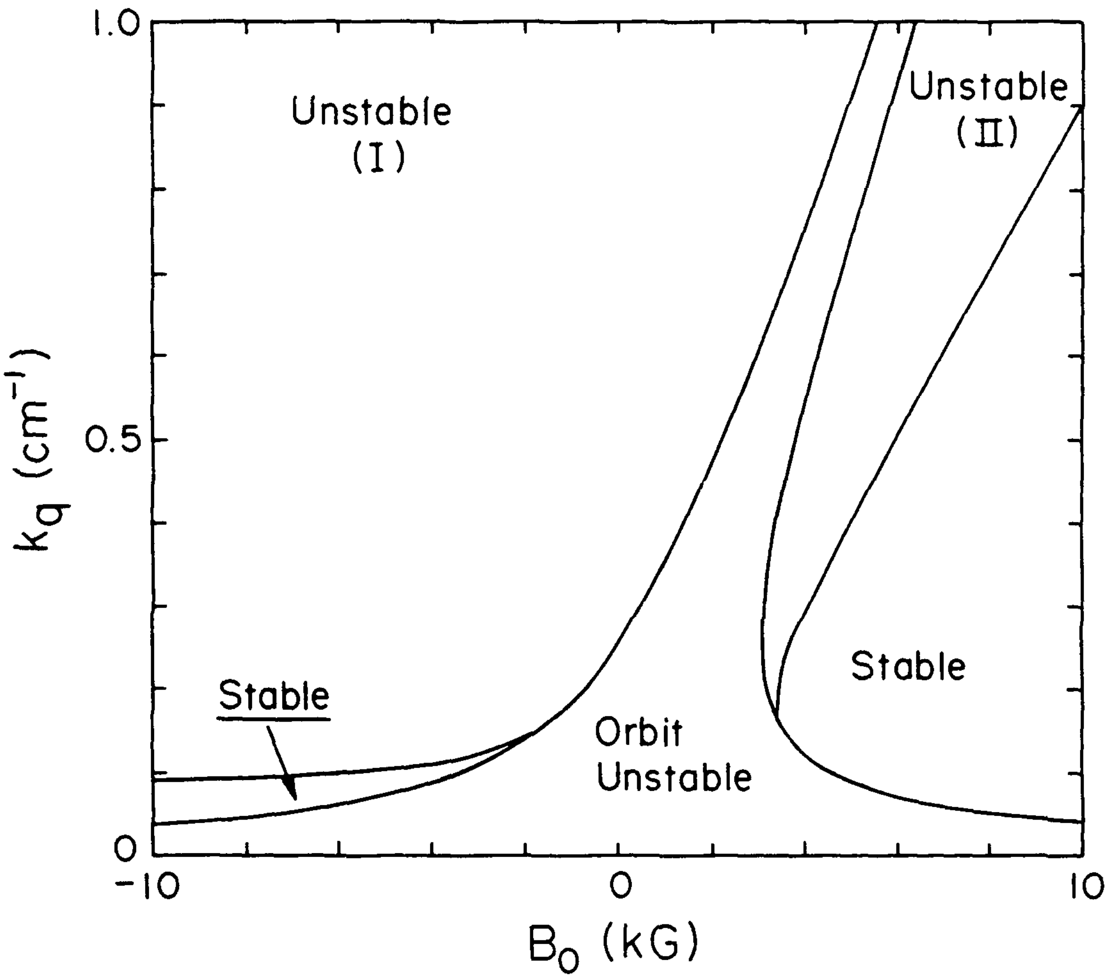

Fig. 1 — Stability diagram for beam energy γ = 7, quadrupole gradient B_q k_q = 200 G/cm and waveguide radius r_g = 3 cm

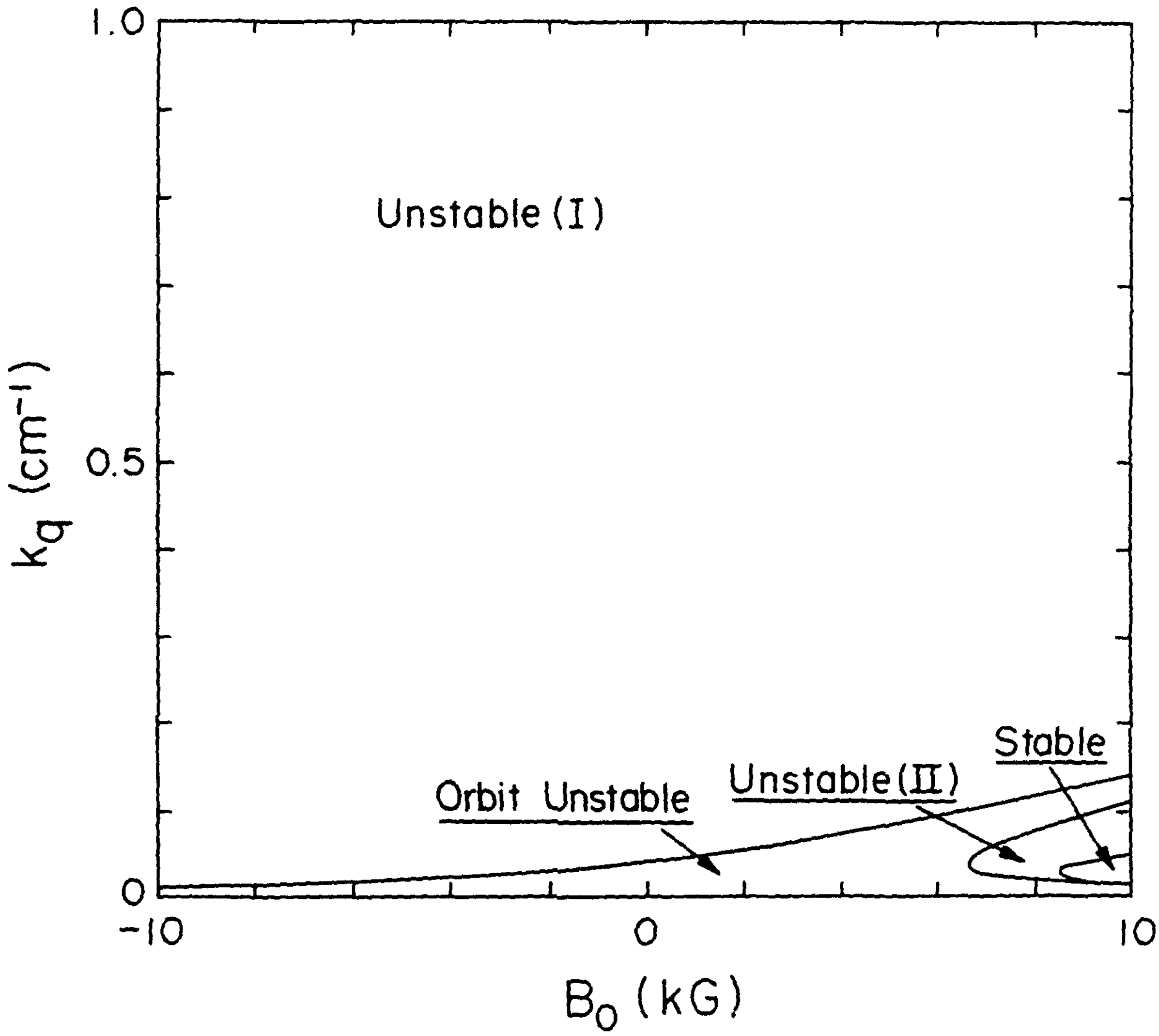

Fig. 2 — Stability diagram for beam energy $\gamma = 90$, quadrupole gradient $B_q\, k_q = 73.4$ G/cm and waveguide radius $r_g = 3$ cm

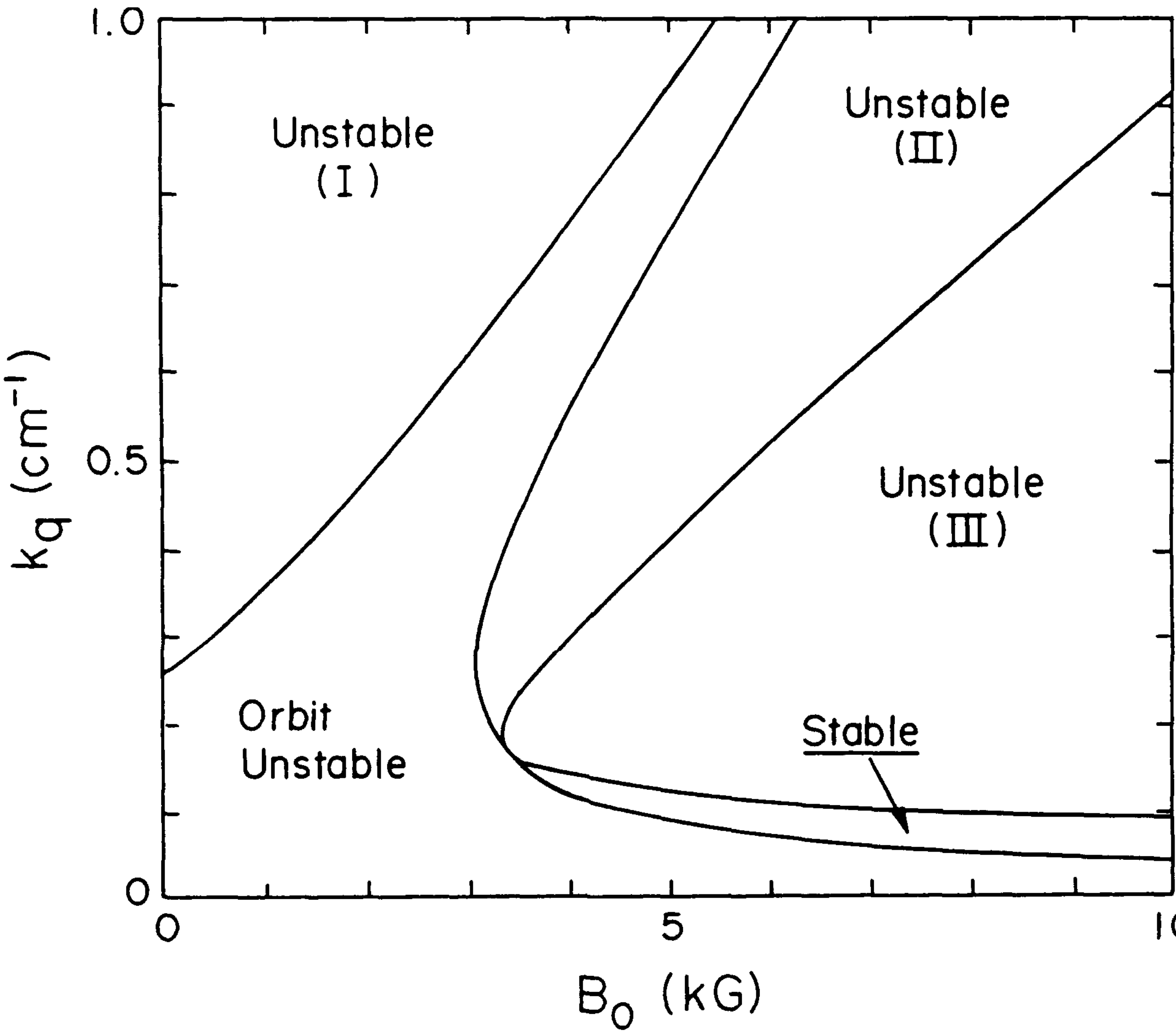

Fig. 3 — Stability diagram for a FODO lattice with beam energy $\gamma = 7$, quadrupole gradient $B_q k_q = 200$ G/cm and waveguide radius $r_g = 3$ cm

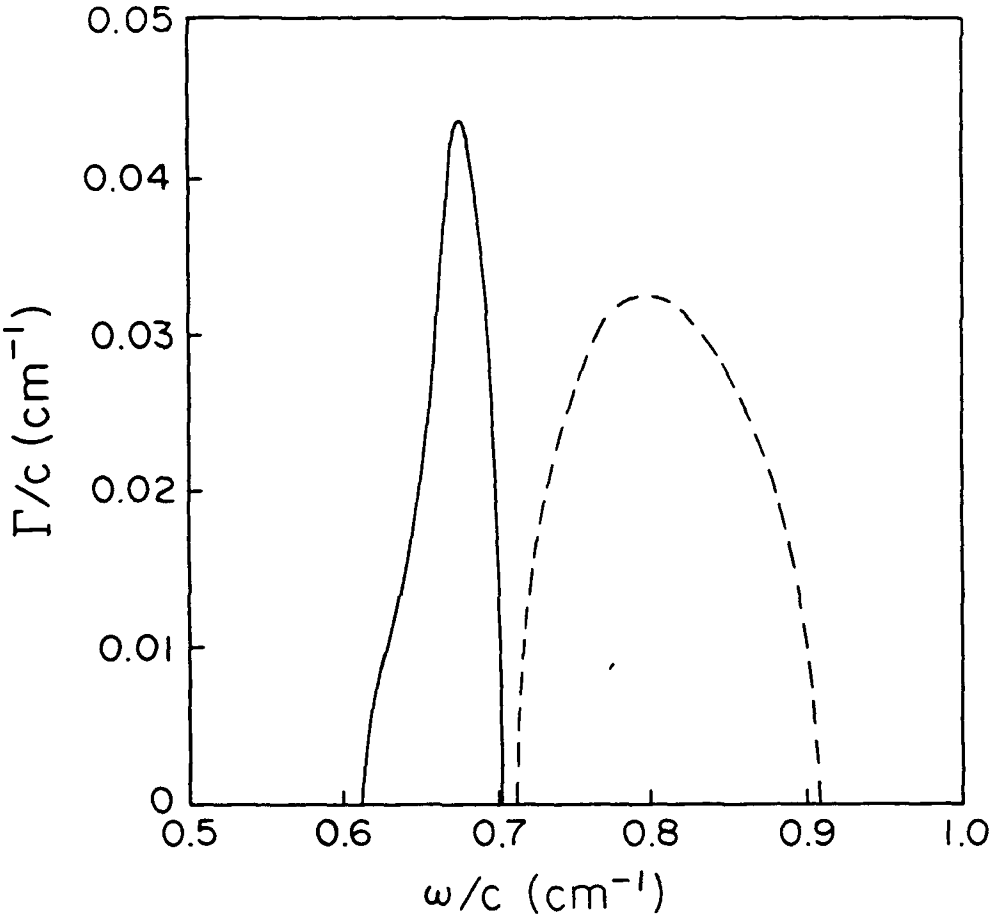

Fig. 4 — Growth rate Γ versus frequency ω for a discrete quadrupole focusing system with $B_o = O$, $B_q K_q = 200$ G/cm, $I_b = 10$ kA, $\gamma = 7$, $r_g = 3$ cm and two different values of $k_q = 0.6\ cm^{-1}$ (solid) and $0.4\ cm^{-1}$ (dashed).